Skyrme-Hartree-Fock calculations of isospin-symmetry breaking

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The isospin-symmetry breaking correction, δ_C, required in the analysis of superallowed Fermi beta decay is discussed in Ref. [1]. For practical reasons, the correction is divided into two components: $\delta_C = \delta_{C1} + \delta_{C2}$, where δ_{C1} incorporates isospin mixing with neighboring states of the same spin and parity and δ_{C2} expresses the difference between the proton and neutron radial functions.

For δ_{C2} we have used both Saxon-Woods radial functions [1] and Hartree-Fock-potential eigenfunctions [2]. The Hartree-Fock calculation is based on the Skyrme zero-range, nucleon-nucleon, density-dependent interaction, for which ten parameters have to be specified. As these parameters are highly correlated in their contributions to nuclear-matter properties, we make no attempt to adjust any of them singly. Rather, our approach is to choose from the more than 400 parameter sets available in the literature a selection that is representative of the field. In this report, we will highlight one of the ten parameters, x_o, which governs the amount of spin exchange – or, equivalently, for an antisymmetric zero-range interaction, the amount of isospin-exchange – in the attractive short-range S-wave part of the interaction. So far, we have considered in detail 16 sets of Skyrme parameters.

For each set of parameters we have computed the δ_{C2} correction for 20 superallowed Fermi decays, from ^{10}C to ^{74}Rb. We add to these values, the δ_{C1} corrections given in Ref. [2] to arrive at the total isospin-symmetry-breaking correction, δ_C. Then these δ_C values were subjected to a test [3] based on the requirement that the δ_C values should lead to corrected Ft values that satisfy the requirements of the Conserved Vector Current (CVC) hypothesis. The test is based on the equation

$$\delta_C = 1 + \delta_{NS} - \frac{K}{ft(1+\delta_R')}, \qquad (1)$$

where ft is the experimental ft-value and δ_{NS} and δ_R' are previously-computed nucleus-dependent contributions to the radiative correction. Thus, to test a set of n calculated δ_C values, we treat K as a simple adjustable parameter and use it to bring the n results from the right-hand side of Eq. (1), which are based predominantly on experiment, into the best possible agreement with the n values of δ_C. The normalized χ^2 (*i.e.* $\chi^2/(n-1)$), minimized by this process, then provides a figure of merit for that set of calculations. Of the three such tests described in [3], we just consider the first test, in which only statistical errors are included on experimental measurements and no errors are included on theoretical quantities. In column two of Table I we list the normalized χ^2 for the 16 sets of Skyrme parameters based on 13 experimental ft values. In all cases, the fit is barely adequate with the normalized χ^2 ranging from 4.9 to 16.1. These values are some distance from the result, 1.2, obtained with Saxon-Woods radial functions. That the Hartree-Fock calculations cannot do better on this test is something of a puzzle.

Table I. Figure of merit, $\chi^2/(n-1)$, for the test [3] discussed in the text and the slope Δ_0 defined in Eq. (2) for 16 Skyrme-potential parameter sets. For comparison, the results with Saxon-Woods functions are shown in the bottom row.

Force	$\chi^2/(n-1)$ 13 data	$\chi^2/(n-1)$ 11 data	Δ_0
SkV	4.92	4.08	1.06
Ska	6.45	3.44	1.01
SGII	8.60	4.09	0.94
SkM*	9.03	4.47	1.00
SkT5	10.40	3.63	1.05
SkRσ	9.87	4.18	0.97
SkGσ	10.32	4.64	0.98
Sly4	10.19	7.45	0.93
SKX	16.12	4.85	0.72
SKXce	14.81	4.97	0.82
KDE0	12.46	6.02	0.77
KDE	12.18	7.36	0.86
SV-min	12.49	4.18	0.87
SV-bas	12.40	4.85	0.86
BSk17	11.33	5.23	0.86
MSL0	9.91	3.44	1.00
Saxon-Woods	1.22	1.39	1.32

The principal defect in the Hartree-Fock results is their failure to obtain a large enough δ_C value for the high-Z cases of ^{62}Ga and ^{74}Rb. Indeed, of the 13 nuclear decays included in the test, the largest contribution to the χ^2 for every choice of Skyrme parameter set comes from ^{62}Ga. Given this difficulty, it is of interest to see how well the Hartree-Fock calculations perform in the lighter nuclei only. Thus in column three of Table I we give the normalized χ^2 values based on 11 experimental ft values, excluding ^{62}Ga and ^{74}Rb. Although the χ^2 values are much better, ranging from 3.4 to 7.5, they are still considerably worse than the 1.4 value obtained with Saxon-Woods radial functions. One of the main contributors to the χ^2 for these Hartree-Fock calculations is ^{26}Al, for which the δ_{C2} values obtained are consistently larger than those obtained with Saxon-Woods radial functions. Another way to quantify this under-prediction for high-Z cases and over-prediction for ^{26}Al is to focus on their difference. We define

$$\Delta_0 = \delta_C(A = 74) - \delta_C(A = 26) \tag{2}$$

and list this quantity in Table I. This slope lies between 0.7 and 1.1 for the 16 Skyrme parameter sets, while the equivalent result from Saxon-Woods radial functions is 30% larger, Δ_0 = 1.32, in agreement with experiment. To date, we have failed to identify any component of the Skyrme force that could be said to be responsible for this defect.

Another curious result is that the correction δ_{C2} computed with Hartree-Fock functions for the decay of a T_z = -1 nucleus depends approximately linearly on the Skyrme parameter, x_o, while in the decay of a T_z = 0 nucleus the δ_{C2} value is independent of x_o. In Fig. 1, we show this dependence for three cases of T_z = -1 emitters, ^{10}C, ^{26}Si and ^{42}Ti, and for three cases of T_z = 0 emitters, ^{26}Al, ^{46}V and ^{62}Ga. We

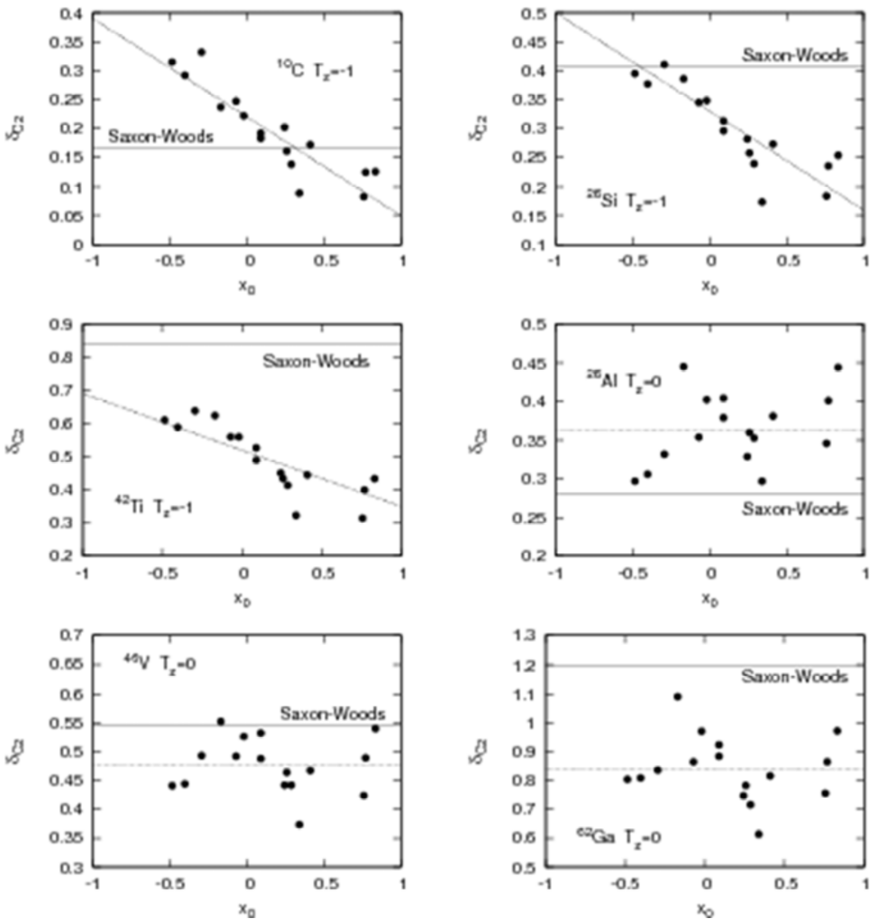

Figure 1. δ_{C2} values in percent units for three T_z = -1 and three T_z = 0 emitters. The black dots are from Hartree-Fock calculations showing the results as a function of the Skyrme asymmetry parameter, x_o. The solid horizontal line is the result obtained with Saxon-Woods radial functions. The dotted line has no physical significance: it is merely to guide the eye. The Skyrme interactions from left to right are: SkGσ, SkRσ, SkT5, SkV, SGII, MSL0, Ska, SkM*, SV-min, SV-bas, SKXce, SKX, BSk17, KDE0, KDE, SLy4.

make the following observations:

- Only the T_z = -1 emitters show a linear dependence on the x_o parameter.
- A best-fit line, linear in x_o, has the same slope for all the T_z = -1 emitters.
- The average δ_{C2} value from the Hartree-Fock calculations over-predicts, relative to the Saxon-Woods calculation, in light nuclei, and under-predicts in heavier nuclei. More specifically in the T_z = -1 emitters, Hartree-Fock over-predicts for ^{10}C, is comparable to Saxon-Woods for ^{14}O, and under-predicts in all heavier cases. For the T_z = 0 emitters,

Hartree-Fock over-predicts for ^{26}Al, is comparable to Saxon-Woods for ^{34}Cl and ^{38}K, and under-predicts in all heavier cases.

- We have found no other correlation between the δ_{C2} value and any of the other 10 Skyrme parameters.

At the time of writing, we have no explanation for these observations, but investigations are continuing.

[1] I.S. Towner and J.C. Hardy, Phys. Rev. C **77**, 025501 (2008).
[2] J.C. Hardy and I.S. Towner, Phys. Rev. C **79**, 055502 (2009).
[3] I.S. Towner and J.C. Hardy, Phys. Rev. C **82**, 065501 (2010).